

# PROBABILISTIC APPROACH TO LIMESTONE PILLAR STRENGTH DETERMINATION

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THE DEPARTMENT OF MINING & MINERALS ENGINEERING  
AT VIRGINIA TECH



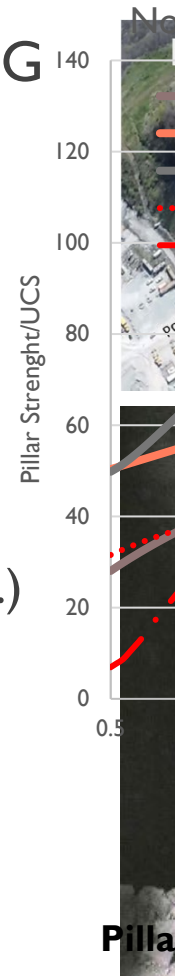
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2. ANALYSIS METHODS FOR PILLAR DESIGN
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# INTRODUCTION

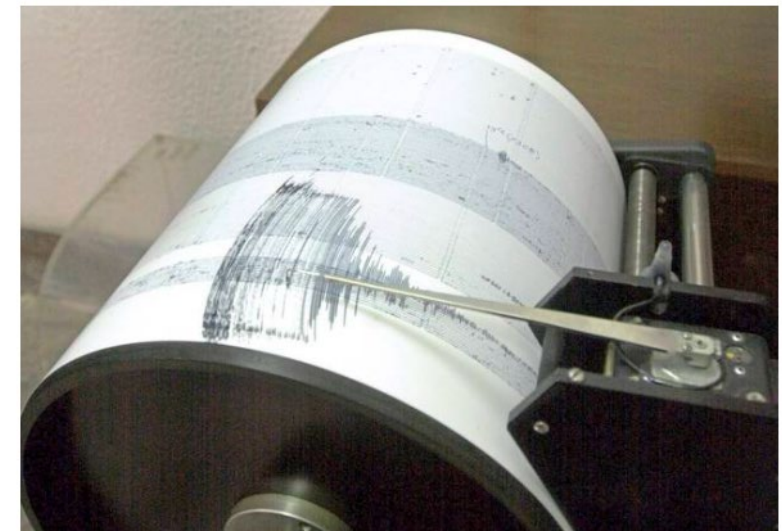
- Pillar design one of the greatest challenges in UG mining
- Pillar failure probability of occurrence vs. consequences
- Pillar design current trends
  - Empirical approaches
  - Site specific conditions (Canada, South Africa, U.S.A.)
  - Neglects failure mechanisms
  - Underestimates uncertainty
- Pillar failure continue to occur...



## Derry Township tremor reclassified as mine collapse



PATRICK VARINE | Tuesday, October 6, 2020 9:50 p.m.



If an earthquake happens and nobody feels it, did it really occur?

Not if it turns out to be a mine collapse.

According to the U.S. Geological Survey, a 2.9-magnitude tremor early Tuesday, centered in an area east of Gray Station Road in Derry Township, was actually the collapse of multiple mine pillars.

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# ANALYSIS METHODS FOR PILLAR DESIGN

## Structurally Controlled Pillar Failure



## Stress Controlled Pillar Failure



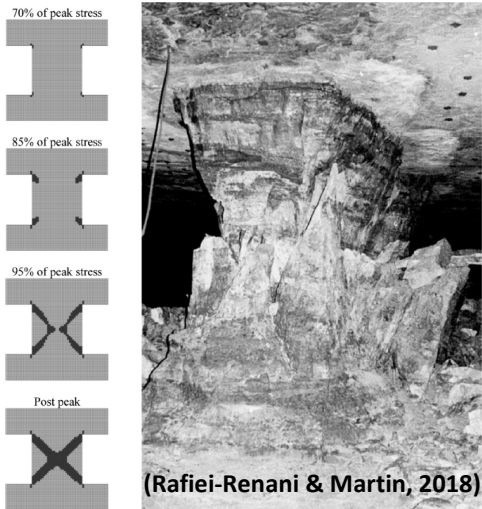
(NIOSH, 2011)



# ANALYSIS METHODS FOR PILLAR DESIGN

$$S_p = K \left( A + B \frac{W^\alpha}{H^\beta} \right)$$

General Form Equation for Pillar Strength Estimation



Continuous Modeling for Pillar Strength Estimation

Estimation of the Average Vertical Stress Applied to the Pillar  $\sigma_p$

Pillar Strength Estimation,  $S_p$

Factor of Safety Calculation, FS

$$F.S. = \frac{S_p}{\sigma_p} > 1$$

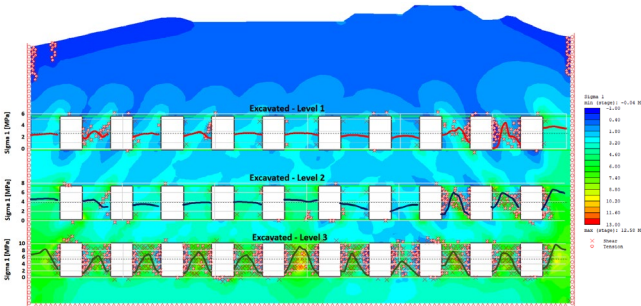
Extraction Ration, ER

$$ER = \frac{\text{Mined Area}}{\text{Tributary Area}} * 100$$

Optimum Solution Safety vs. Recuperation

Monitoring

$\sigma_p = \gamma h * Tributary Area$   
Pillar Stresses through the Tributary Area Theory



Continuous Modeling for Pillar Stress Estimation



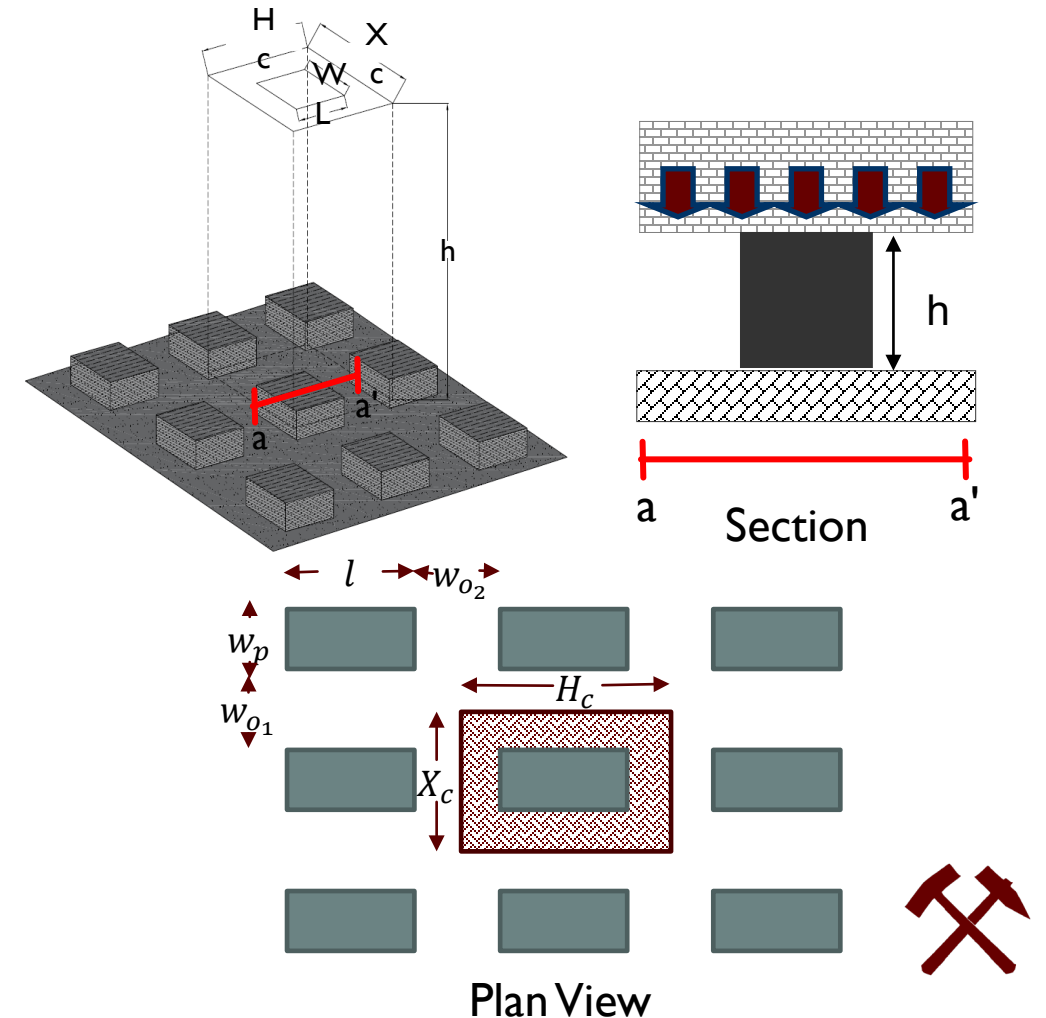
# ANALYSIS METHODS FOR PILLAR DESIGN

## Analytical Methods

- Theoretical approaches are derived from mathematical expressions based on the static equilibrium principle.
- Describe the performance of mine pillars subject to a load for a given set of input variables.
- These methods are subject to a series of assumptions and limitation.
- The most common analytical method used in pillar design is the tributary area for estimating pillar load and its derivations.

$$\sigma_p = \frac{\rho g h (X_c * H_c)}{(w_p * l)}$$

↗ Tributary Area  
↘ Pillar Area



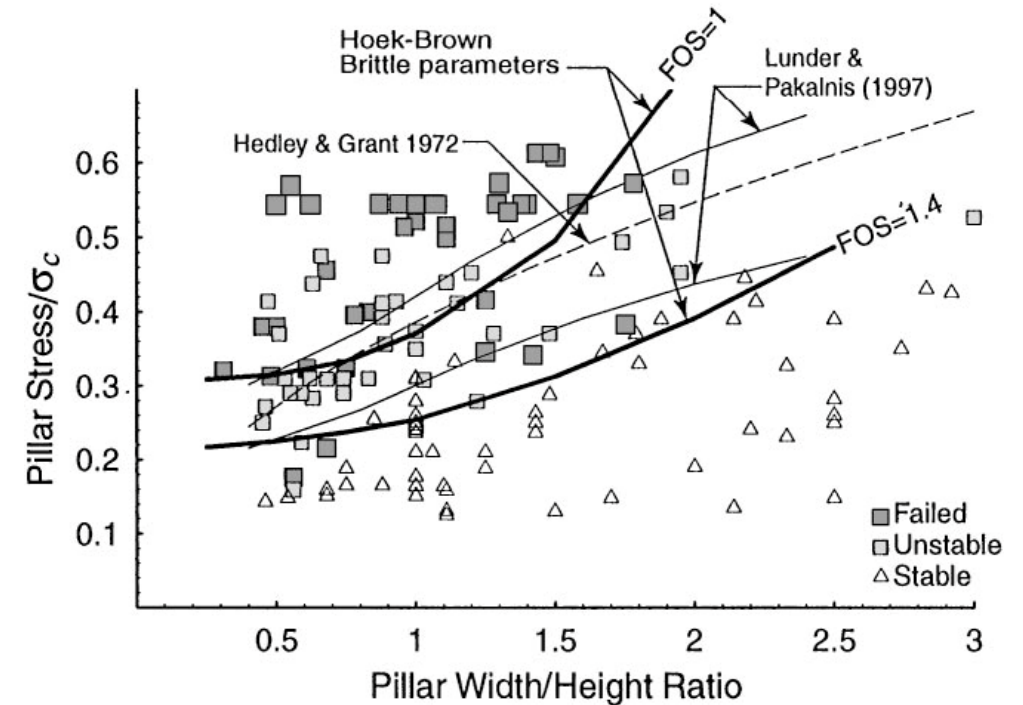
# ANALYSIS METHODS FOR PILLAR DESIGN

## Empirical Methods

- Empirical equations are derived from back analysis of stable, unstable and failed pillar cases.
- Consider parameters: Intact rock strength (UCS) and pillar dimensions (Width and Height)
- Developed based on specific case studies, and each considers different geological environments, rock types, structural settings, and mining methods.
- Although, there have been multiple proposed models, all the equations have the following form:

$$S_p = K \left( A + B \frac{W^\alpha}{H^\beta} \right)$$

**General Form Equation for  
Pillar Strength Estimation**



# ANALYSIS METHODS FOR PILLAR DESIGN

## Empirical Methods

Resistencia del Pilar	Tipo de Roca	Autor
$S_p = 0.578 * UCS \left( \frac{W^{0.5}}{H^{0.75}} \right)$	Quartzites	Headley-Grant, 1972
$S_p = 0.354 * UCS \left( 0.778 + 0.222 \frac{W}{H} \right)$	Limestone	Krauland-Soder, 1987
$S_p = 0.44 * UCS(0.68 + 0.52\kappa)$ $\kappa = \tan \left( \cos^{-1} \left( \frac{1 - C_{pav}}{1 + C_{pav}} \right) \right)$ $C_{pav} = 0.46 * \left[ \log \left( \frac{W}{H} + 0.75 \right) \right]^{\frac{1.4}{W/H}}$	Hard Rock – Canada	Lunder-Pakalnis, 1997
$S_p = 0.65 * UCS * LDF * \frac{W^{0.3}}{H^{0.59}}$ $LDF = 1 - DDF * FF$	Limestone	Esterhuizen et al., 2011 (NIOSH)

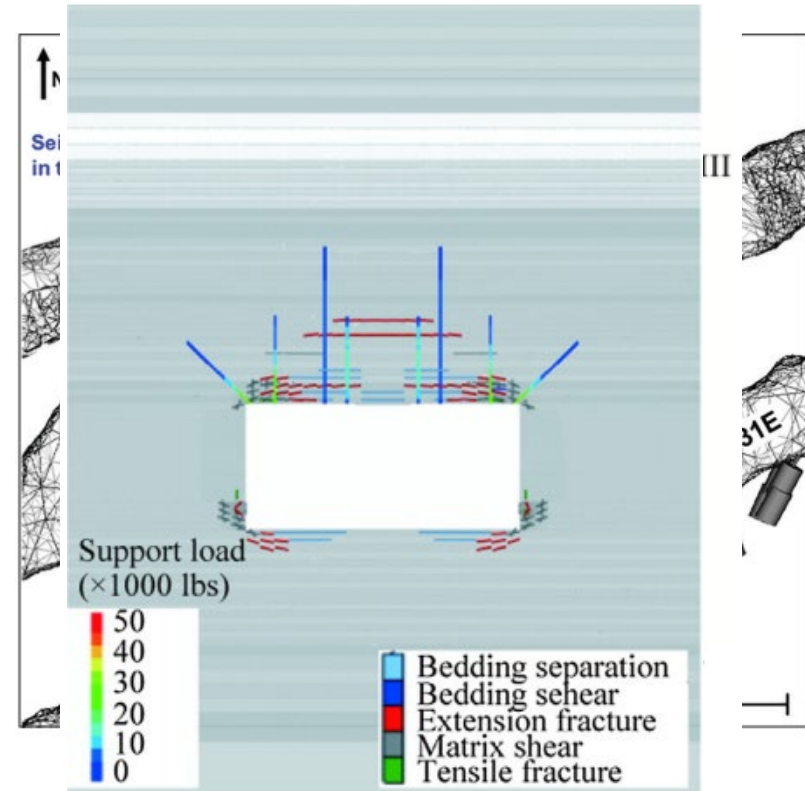




# ANALYSIS METHODS FOR PILLAR DESIGN

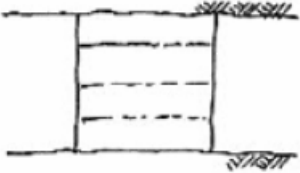
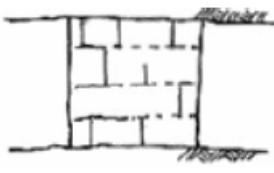
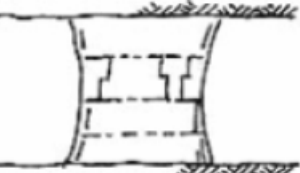

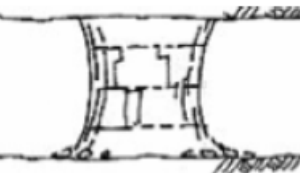
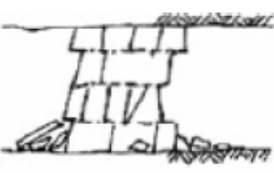



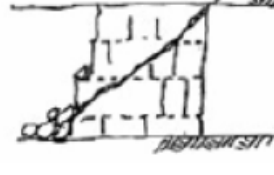
## Observational Methods

- more than a design approach is a verification method in which field engineers can verify if the condition of the pillar is in accordance with the design (Stille & Holmberg, 2008).
- Instrumentation methods to determine stress redistribution after the excavation, and measure rock deformation. To validate and calibrate numerical models, or monitor hazards.
- Include visual rating systems that have been proposed to evaluate the conditions of mine pillars.



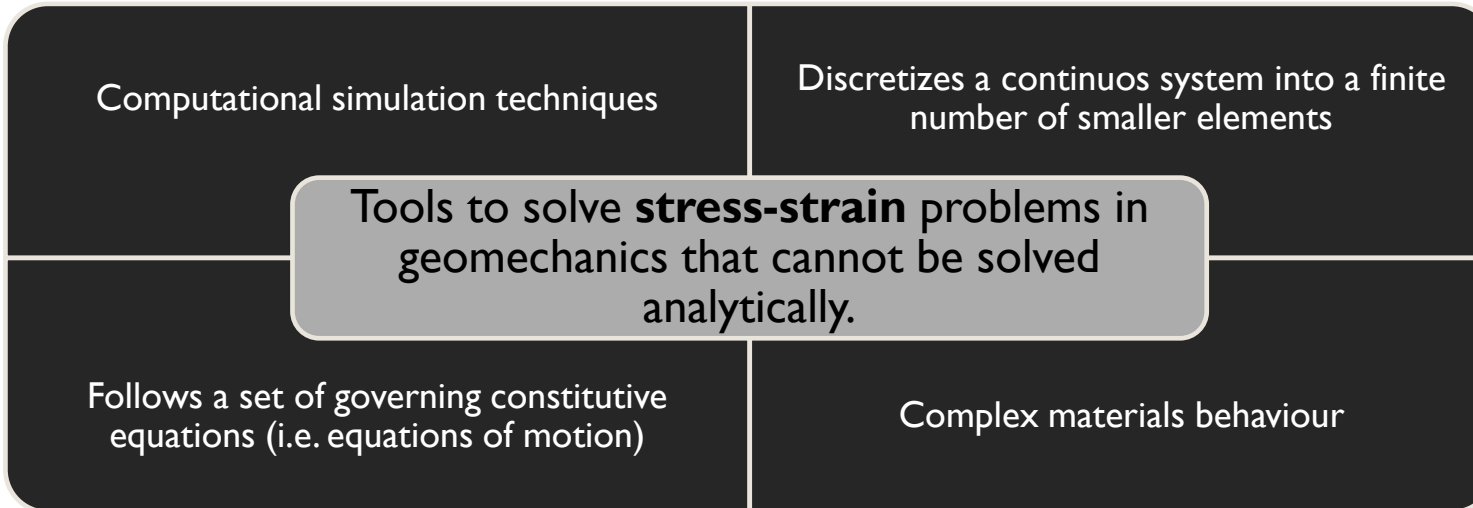
(Gangrade et al., 2019)



a) Pillar Stress Rating			b) Geological Structure Rating		
Rating	Sketch		Rating	Sketch	Description
1 None		No stress related fracturing or spalling observed. Joint or blast related damage may exist.	1 None		Less than 0.3 m (1 ft) of joint related fallout during blasting. Blast damage may exist.
2 Minor		Minor slabs or spalling, fractures through intact rock at corners, pillar corners and walls may be concave, does not typically deteriorate after initial mining and scaling.	2 Minor		Pillar shape affected by 0.3- 1 m (1-3 ft). Some joint or bedding fallout during blasting, may form step at bedding planes. No or little further fallout after initial scaling.
3 Moderate		Slabbing, onion-skin, fractures more than 1 m long, joints opened, corner damage, pillars may need re-scaling after initial development. Original square pillar shape maintained.	3 Moderate		Pillar shape affected by 1-3 m (3- 10 ft). Joint or bedding controlled fallout. Fallout can continue after initial mining and scaling.
4 Severe		Spalling to hourglass shape. Open cracks in pillar more than 1 m long, debris around pillar, original square shape of pillar no longer visible, saw tooth slabs on ribs	4 Severe		Large block fallout >3 m (>10 ft). Pillar shape compromised by large block extrusion or block sliding on steep plane. Falls continue after initial mining and scaling.
5 Very Severe		Formation of large open cracks, extreme hourglass. Pillar likely lost most of its residual strength.	5 Very Severe		Pillar bisected by through-going structure dipping at more than 35 degrees. Potential or actual loss of top half of pillar. Pillar strength depends on discontinuity length.

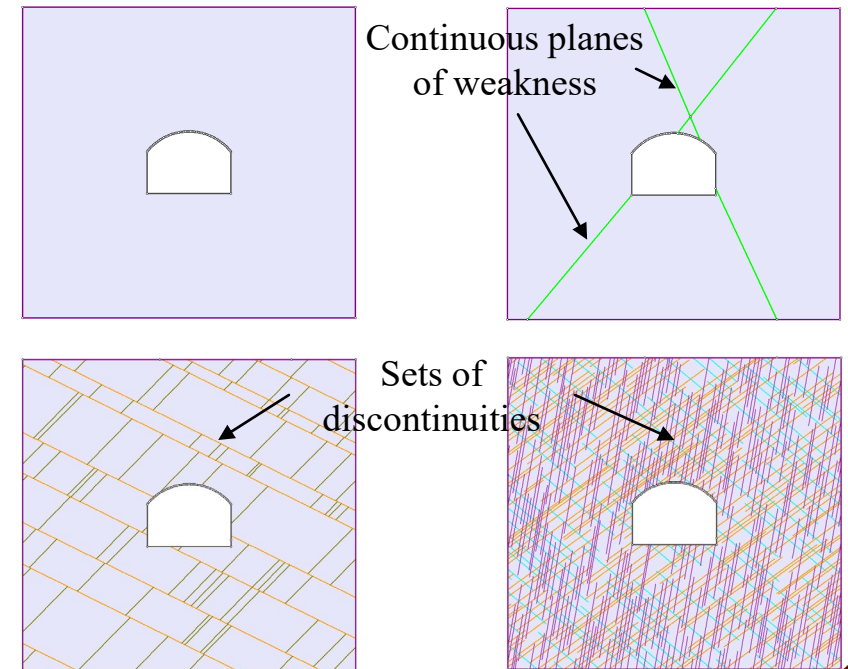
# ANALYSIS METHODS FOR PILLAR DESIGN

## Numerical Methods



- Estimate the stresses acting on the pillars
- Evaluate pillar failure mechanisms given a constitutive model to predict the material failure

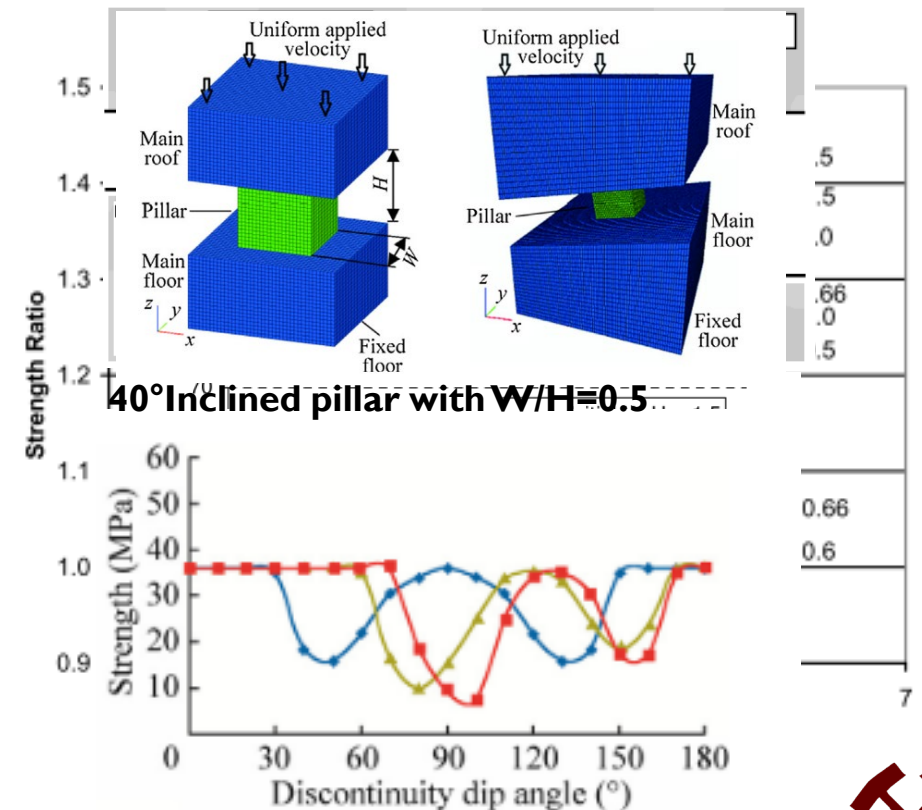
## Continuous vs Discontinuous Modeling



# ANALYSIS METHODS FOR PILLAR DESIGN

## Continuous Methods in Pillar Design

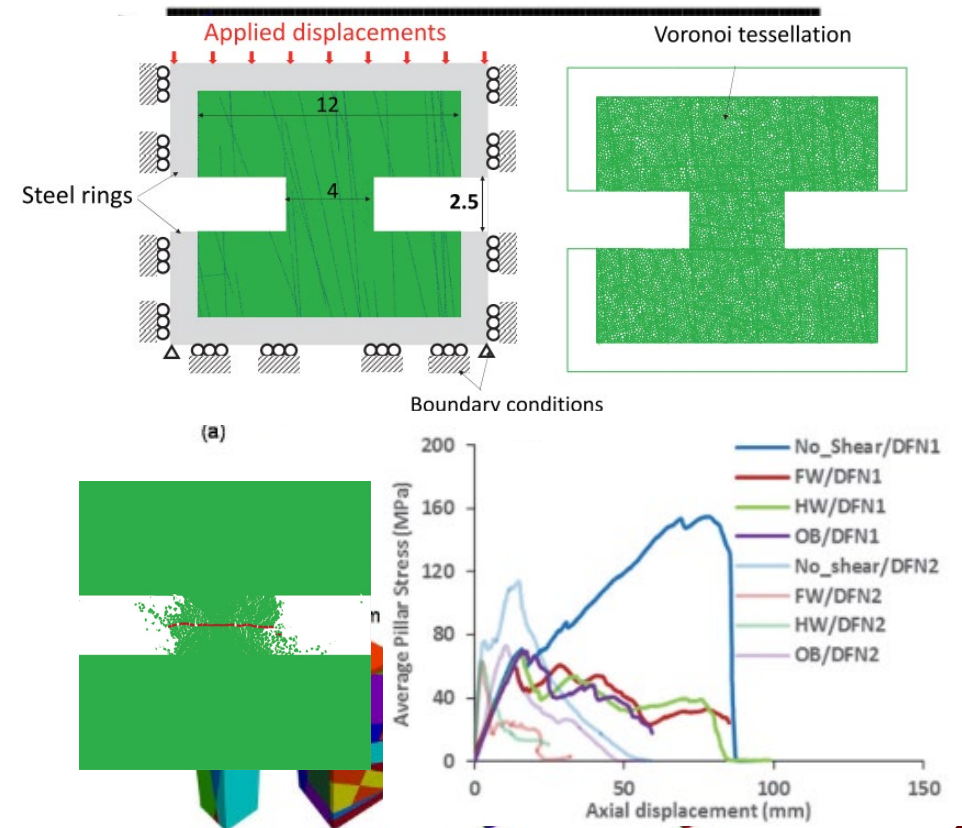
- Determine the effect of weak bands in pillar strength in underground limestone mines.  
(Esterhuizen & Ellenberger, 2007)
- Evaluate the Impact of length on Pillar Strength.  
(Dolinar & Esterhuizen, 2007)
- Evaluate the effect of benching around pillars.  
(Esterhuizen, Dolinar, & Ellenberger, 2007)
- Evaluate the effect of discontinuities on inclined pillar's strength.  
(Jessu & Spearing, 2019)



# NUMERICAL METHODS FOR PILLAR DESIGN

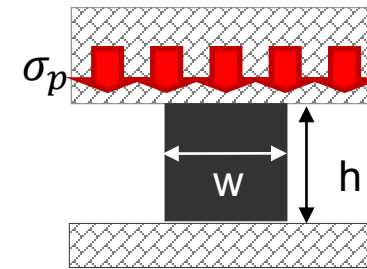
## Discrete Methods in Pillar Design

- Hybrid FEM/DEM model with DFNs to simulate slender fractured hard rock pillars. (Elmo & Stead, 2010)
- PFC3D with DFNs to evaluate peak strength and post-peak strain softening properties of a series of jointed pillar. (Zhang et al., 2015)
- Guidelines on dealing with moderately fractured rock masses by integrating LiDAR, DFN generation, and numerical simulations. (Vazaios et al., 2018)
- Estimated fractured pillar strength intersected by a clay filled structure. (Muaka et al., 2017)



# RISK ANALYSIS APPROACHES IN PILLAR DESIGN

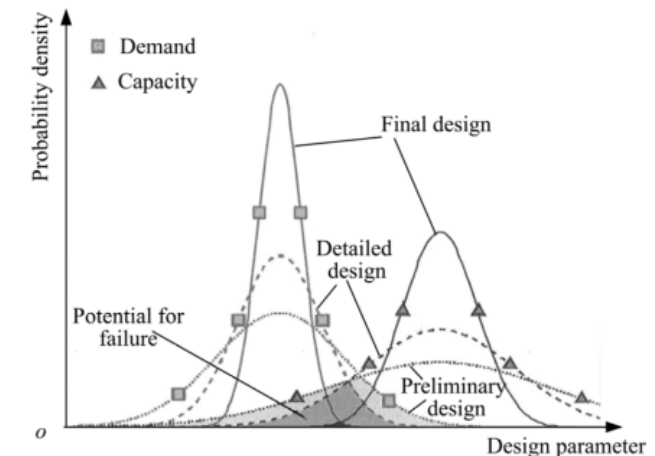
- **Risk analysis** is the process through which an understanding of all the possible risks that can arise during a particular stage of a process is developed
  - provides inputs during decision-making stages in an engineering project to define which risks should be prioritized and addressed
  - considers the **source of the risk**, its **consequences**,
- **Probabilistic Risk Analysis (PRA)**: consists of assessing probability density functions (PDFs) of design parameters, such as loads and capacity for a certain system, and computing from these estimates the probability of failure.



$$F.S. = \frac{\text{Strength}}{\text{Stress}}$$

$$\text{Failure} = \{F.S. < 1\}$$

$$PoF = P(F.S. < 1)$$



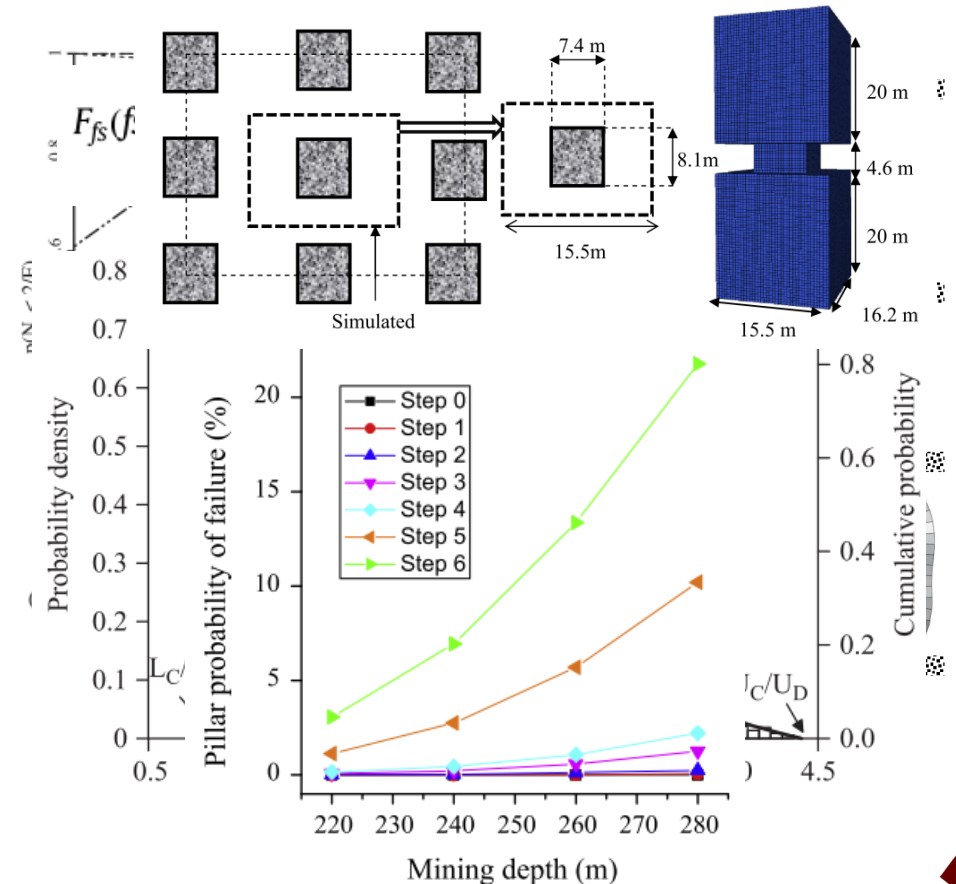
“... Unfortunately, this type of analysis (PRA) is not possible for one of the most important groups of problems in underground excavation engineering, i.e., those problems involving stress driven instability...”



# RISK ANALYSIS APPROACHES IN PILLAR DESIGN

## Probabilistic Risk Analysis in Pillar Design

- Assessed the influence of spatially varying strength in the stability of pillars by combining random field theory with FEM. (Griffiths et al., 2002)
- Applied reliability theory in common stability problems in underground mine design. (Nomikos & Sofianos, 2011)
- Integrated numerical modeling results with Artificial Neural Networks to evaluate pillar probability of failure. (Idris et al., 2015)



# RISK ANALYSIS APPROACHES IN PILLAR DESIGN

## Probabilistic Risk Analysis in Pillar Design

- Used a Monte Carlo approach to evaluate the impact of measured pillar size variability on the mine design.

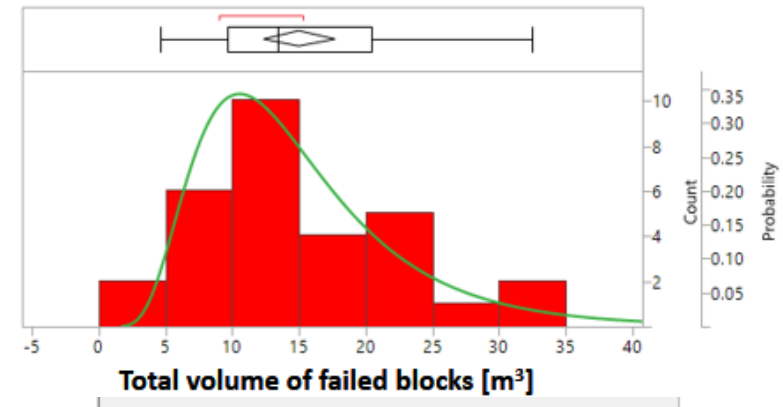
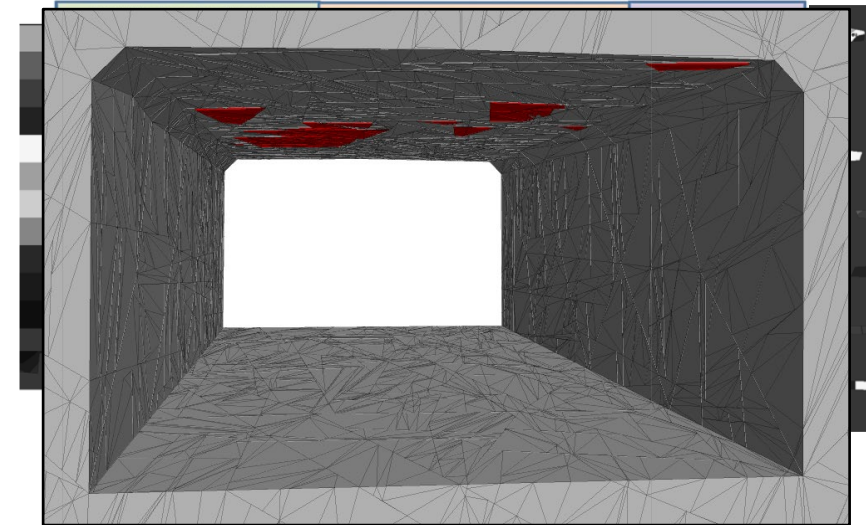
(Walls et al., 2015)

- Proposed and implemented a risk evaluation model to quantify the expected injuries and economic losses resulting from rock falls in underground mines in south Africa

(Joughin et al., 2015)

- Integrated LiDAR, DFN generation, stochastic DEM to predict rock fall probability in an underground stone mine.

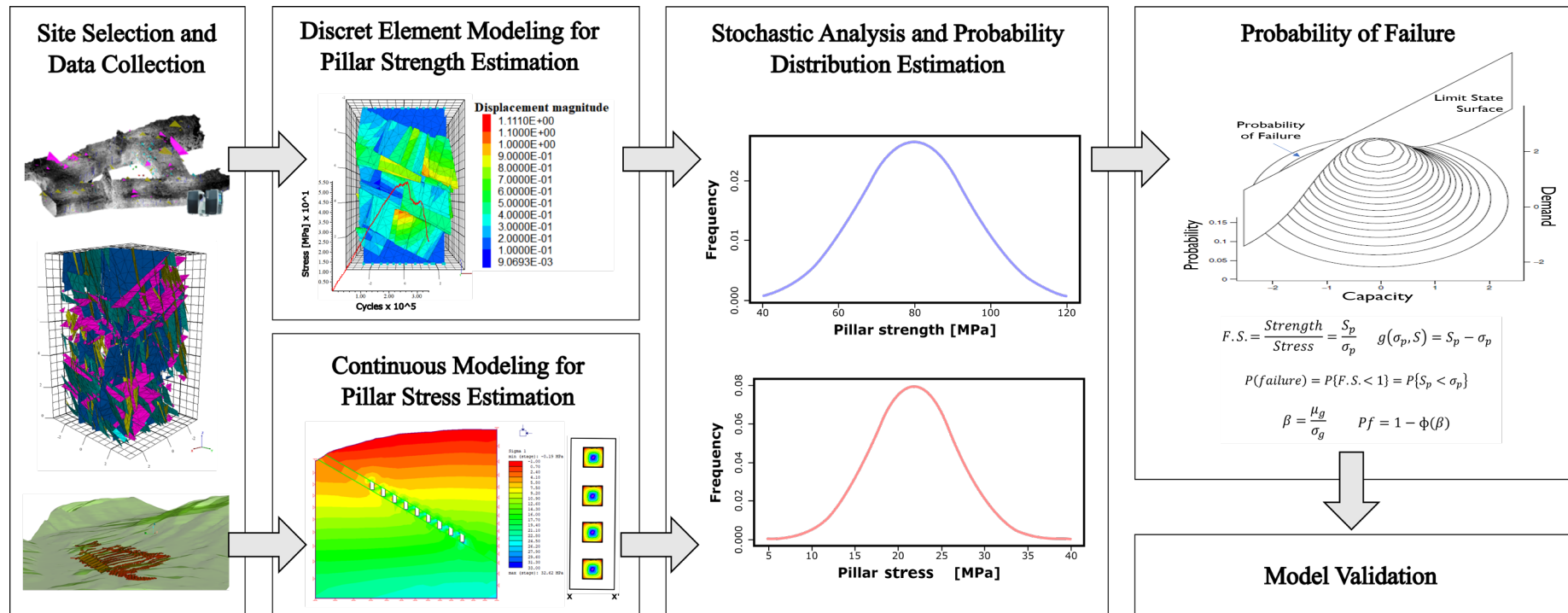
(Monsalve et al., 2019)





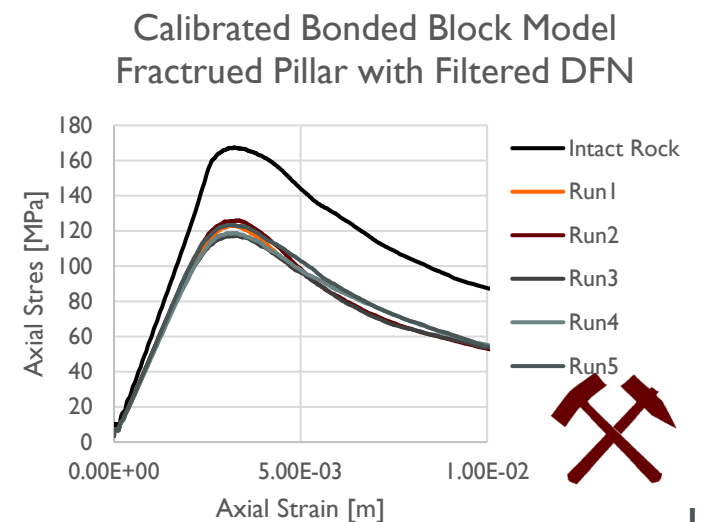
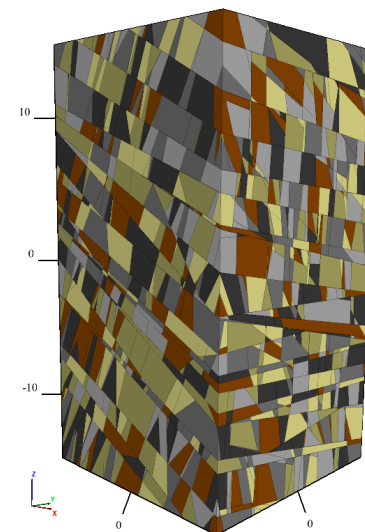
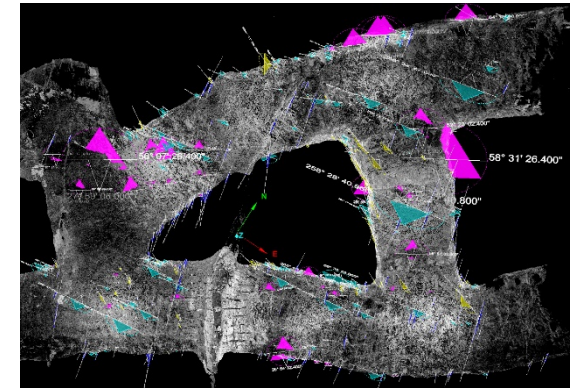
# OBJECTIVE AND METHODOLOGY

- The goal of this work is to develop a framework to estimate the pillar probability of failure based on the stochastic discrete element modeling approach for pillar strength determination and finite volume continuous modeling for stress estimation, which could be globally implemented by considering site-specific conditions of each operation.

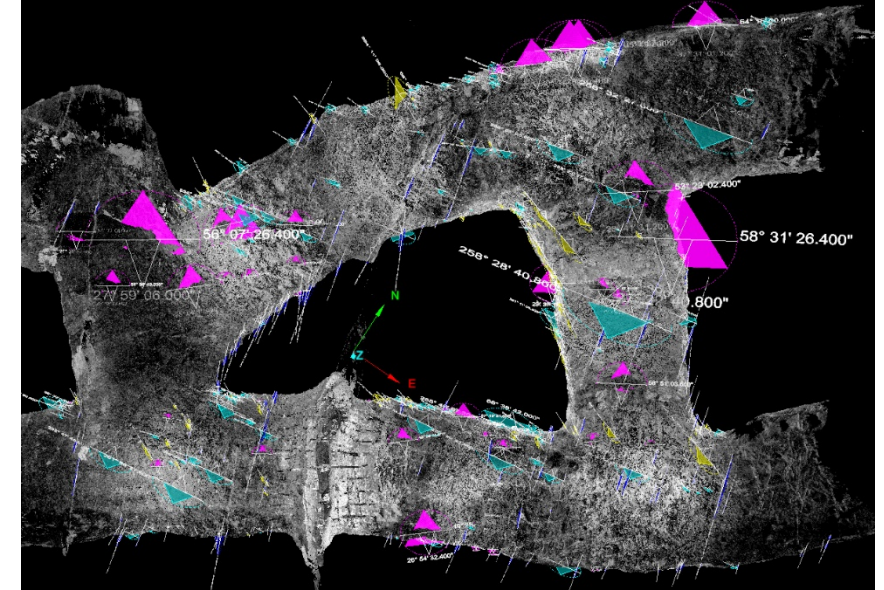
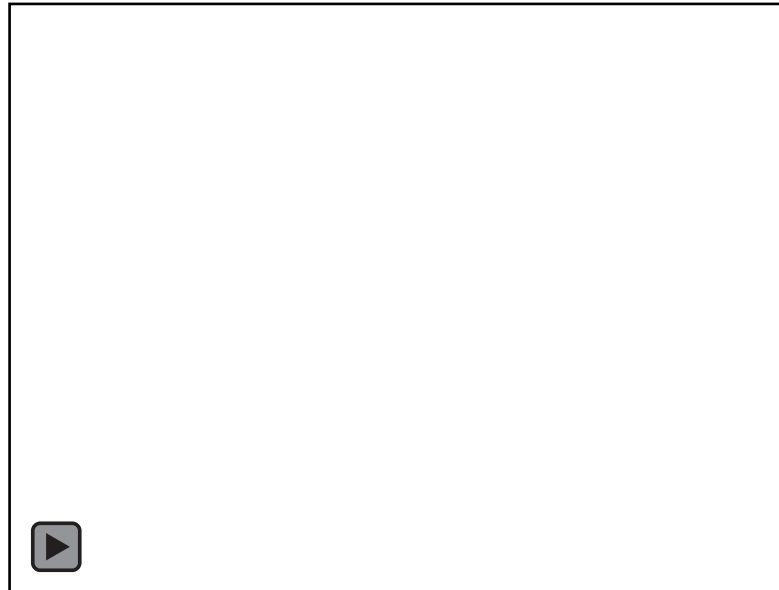


# METHODOLOGY

1. Field Data Collection and Geomechanical Characterization
2. Discrete Fracture Network (DFN) Definition
3. Discrete Element Modeling Approach Selection
4. Model Calibration and Validation
5. Stochastic Discrete Element Modeling



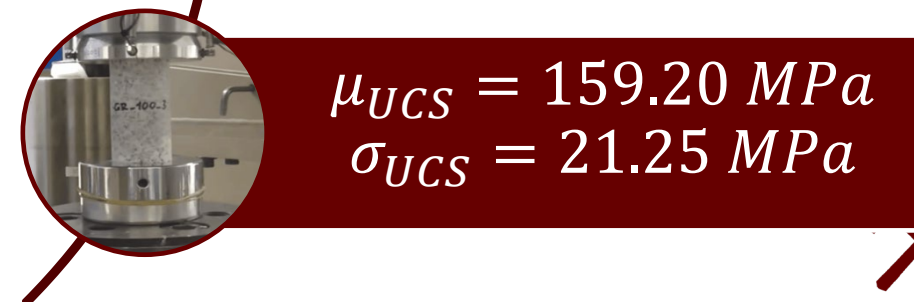
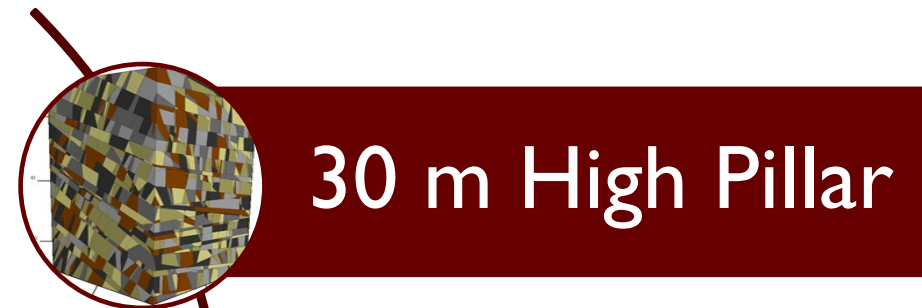
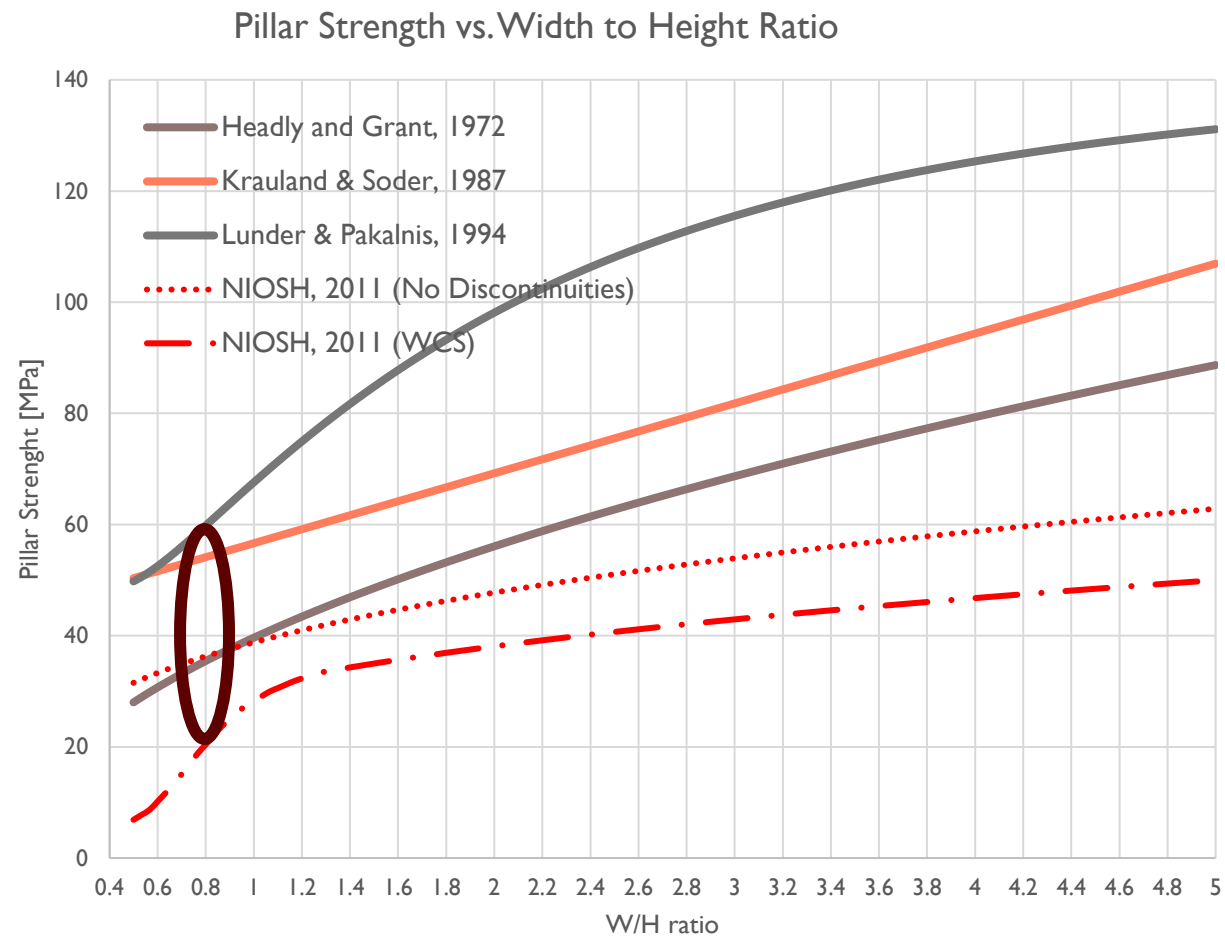
# CASE STUDY MINE



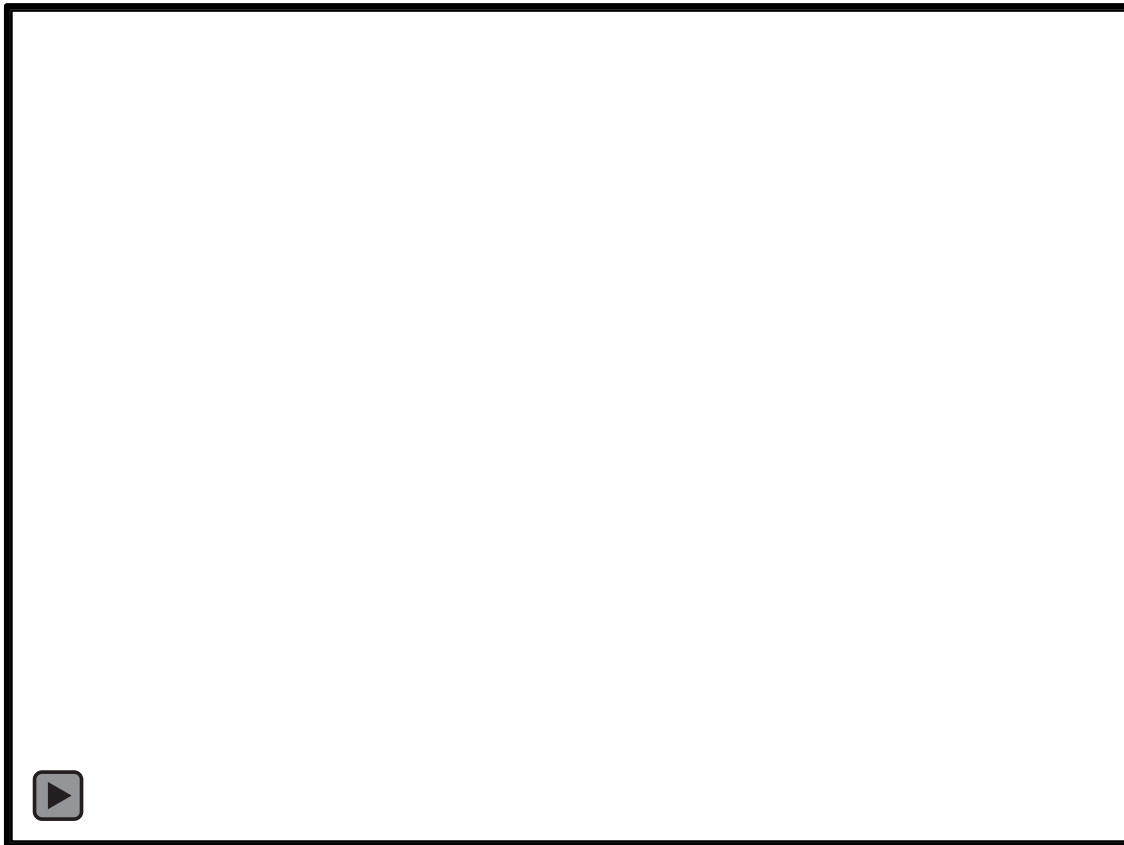
- 30° Dipping deposit
- 30 m (100 ft) thick seam
- Room & Pillar mining method with eventual stoping
- Structurally controlled instability main ground stability hazard
- 24 m x 24 m pillars (80 ft x 80 ft)
- Stope height  $\approx$  30 m (100 ft)
- Drifts 12.8 m x 7.6 m (42 ft x 25 ft)



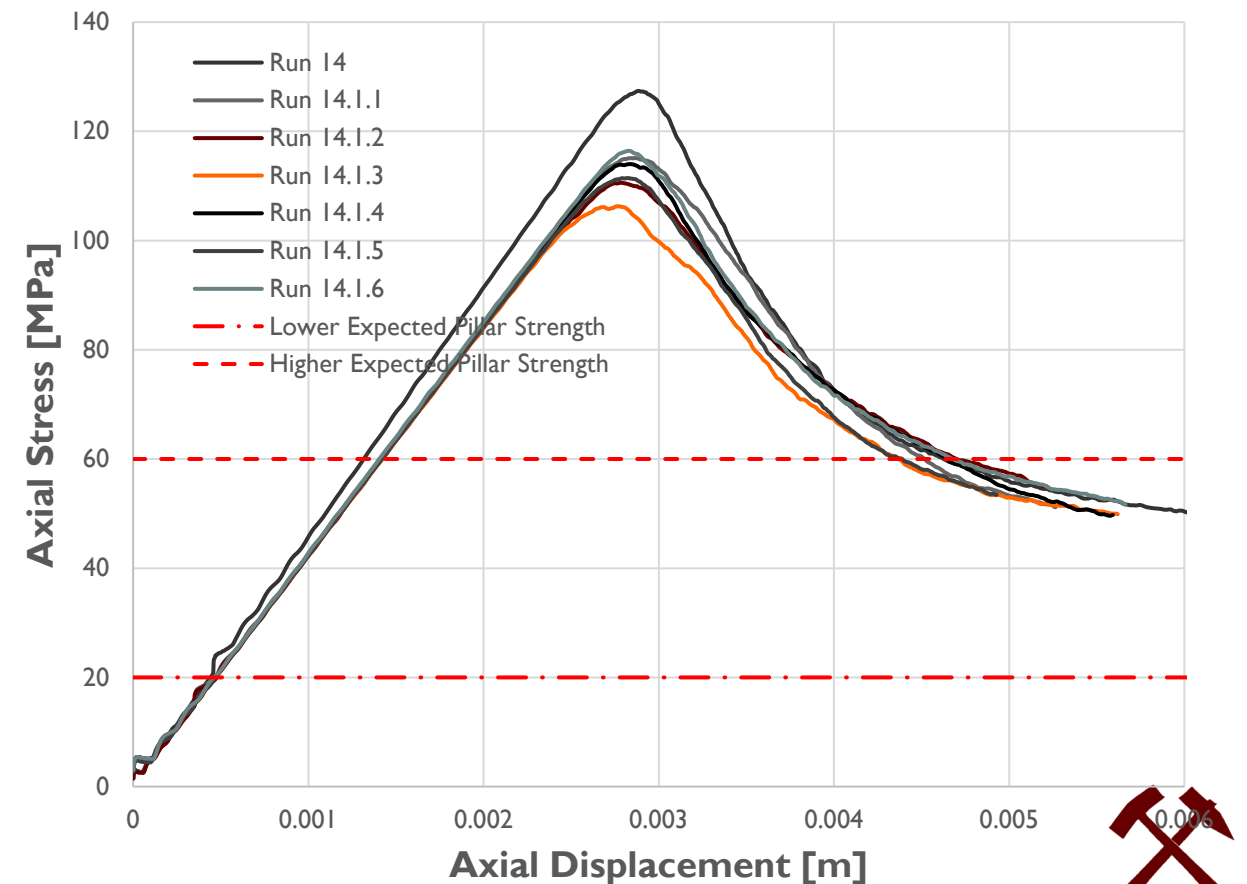
# CASE STUDY PILLAR STRENGTH ESTIMATION



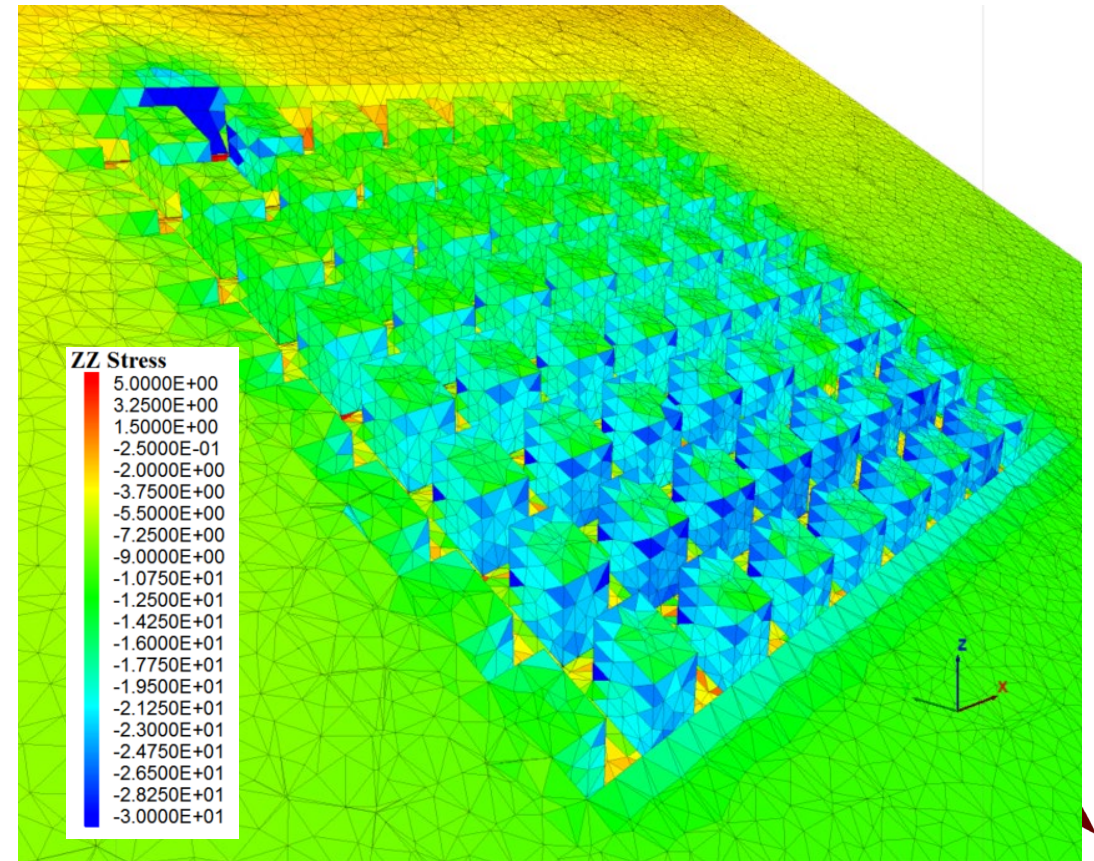
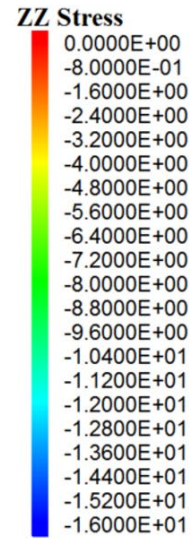
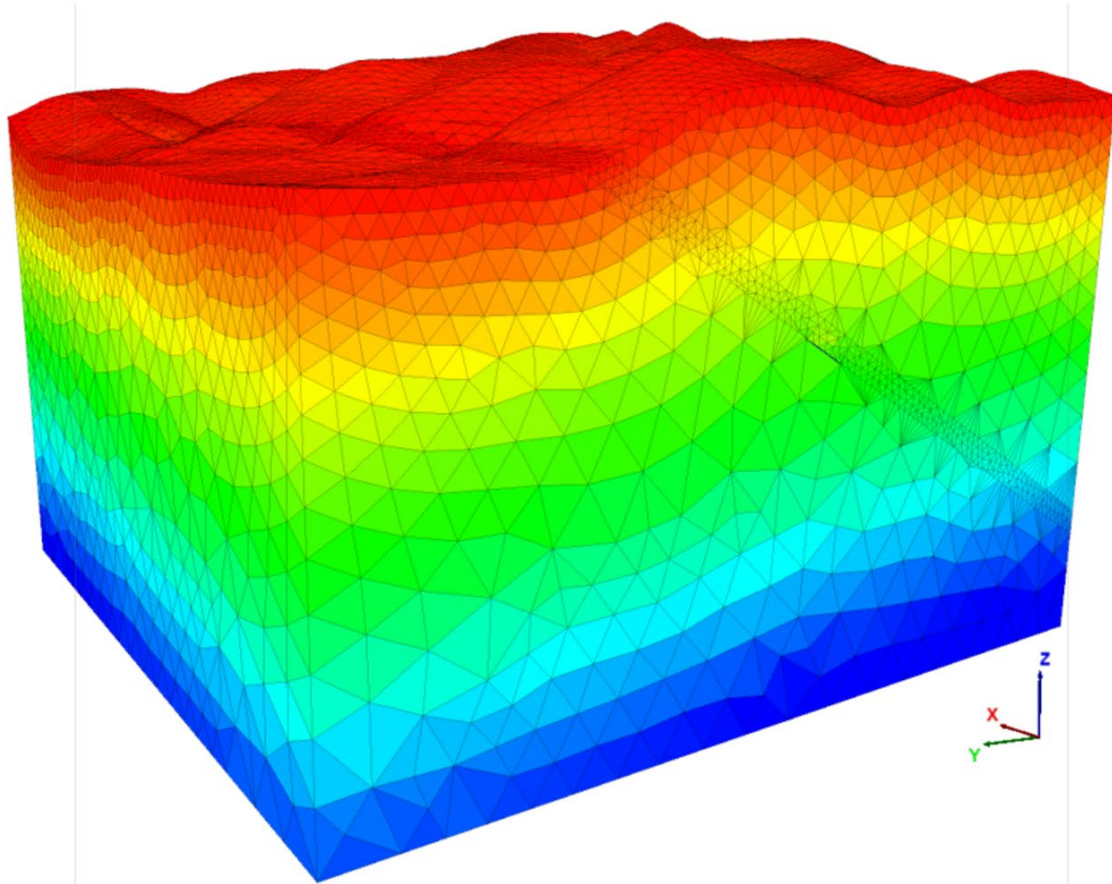
# PILLAR STRENGTH ESTIMATION PRELIMINARY RESULTS



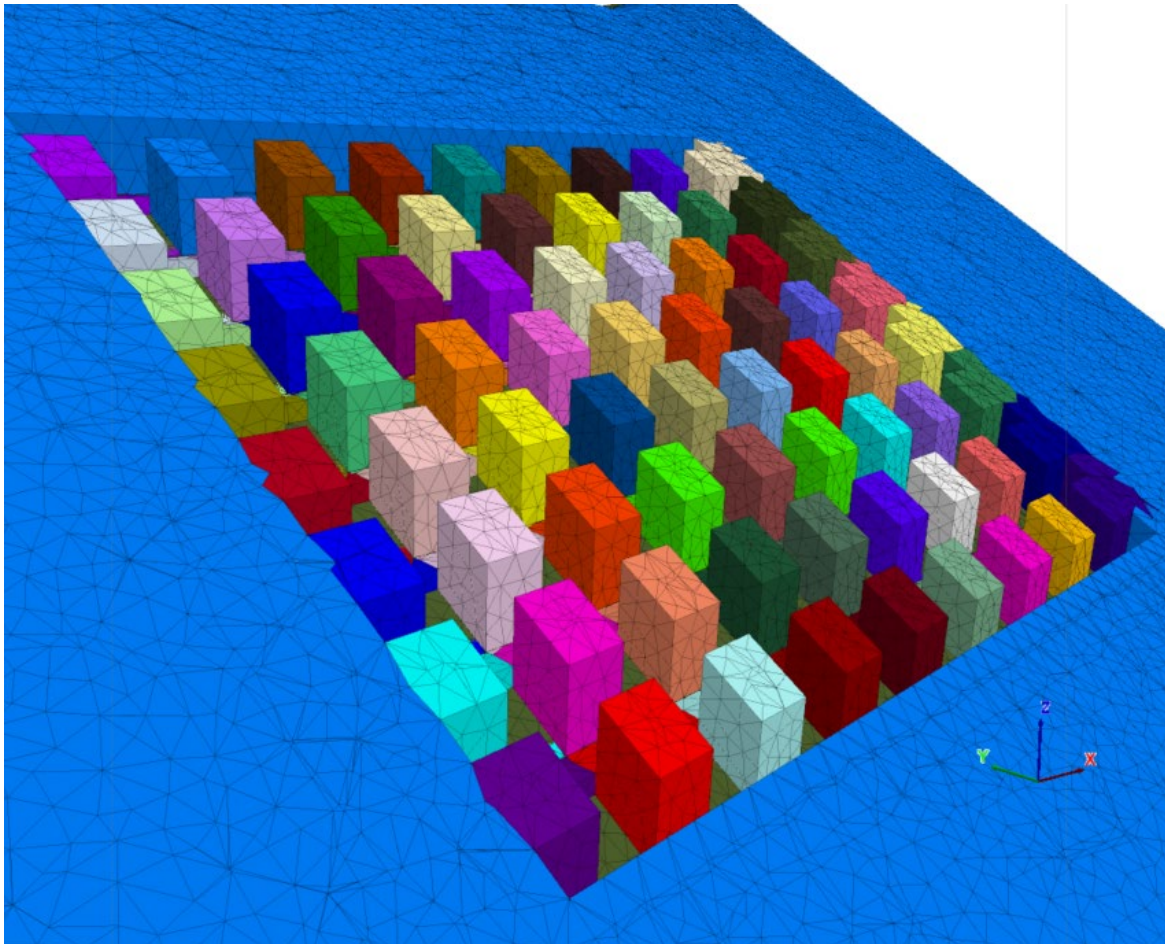
### Stochastic Fractured Pillar Strength Results



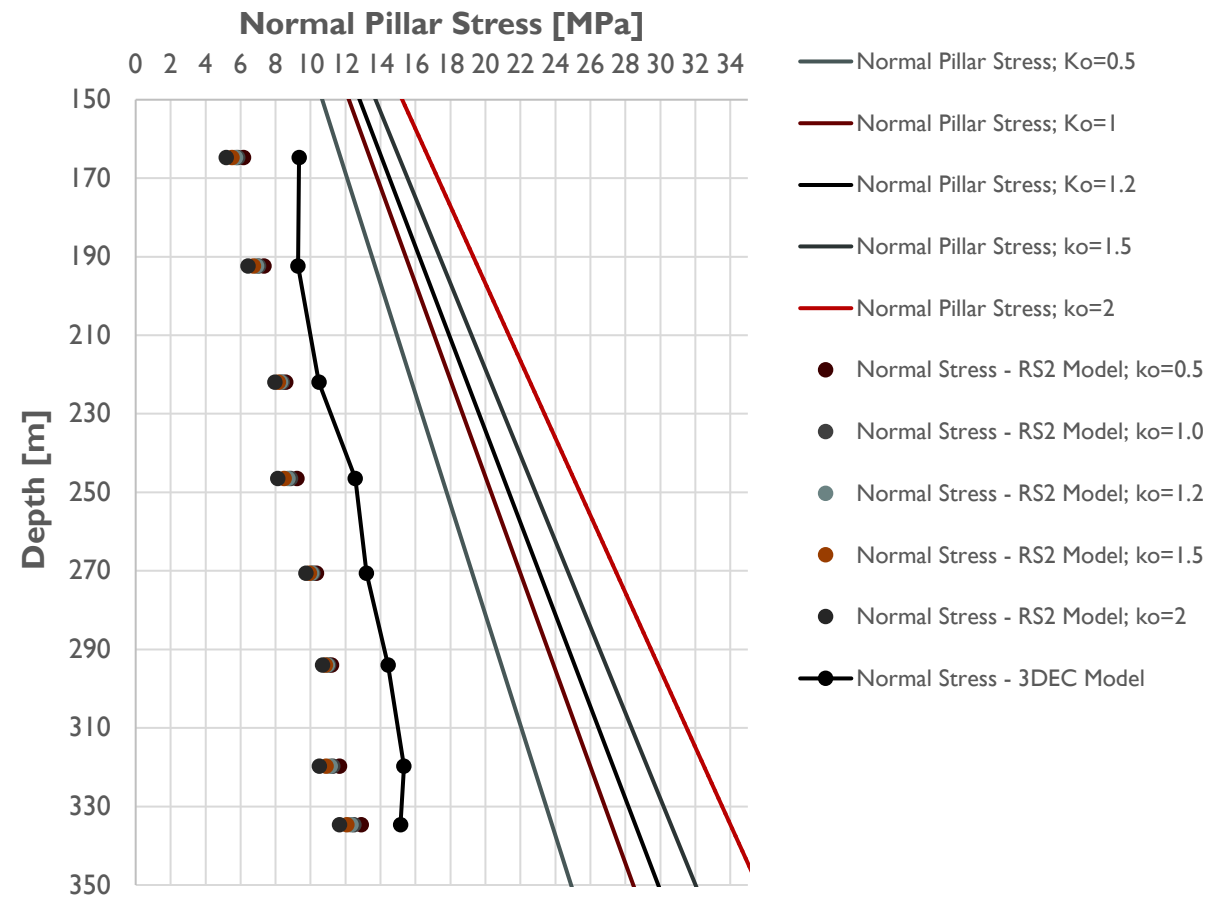
# 3D PILLAR STRESS ESTIMATION



# 3D PILLAR STRESS ESTIMATION



## Normal Pillar Stress vs Depth for a 30° Dipping Deposit



# CONCLUSIONS

- All empirical, analytical and numerical design methods have underlying assumptions and limitations.
- Numerical models validated and calibrated with instrumentation have gained acceptability throughout the years in pillar design.
- Recent technological advances have enabled probabilistic risk analysis approaches to be implemented into pillar design.
- A risk-based methodology that integrates stochastic discrete element modeling and finite volume modeling to estimate pillar stress and strength was presented.





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THANK YOU!

